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Equations of Motion for the Perturbed Restricted Three-Body Problem

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BROWN and Shook[1] discussed a particular case of the perturbed restricted problem in their treatment of the motion of the Trojan asteroids. But they did not give a general method for the perturbed restricted problem. Farquhar[2] has given such general equations of motion. In the unperturbed circular problem, one normalizes to unity the (constant) distance between primaries r, their mean motion $\dot{\theta}$, and the sum of their masses. The smaller primary is of mass μ. Also, the coordinate system (x, y, z) rotates with angular velocity $\dot{\theta}$, such that the locations of masses μ, $1-\mu$ are, respectively, $x_1 = -(1-\mu)$, $x_2 = \mu$. To incorporate the indirect effect in the perturbed case, Farquhar has

$$r = 1+\rho(t), \quad \dot{\theta} = 1+v(t) \tag{1}$$

so that the primaries are at $x_1 = -(1-\mu)(1+\rho)$, $x_2 = \mu(1+\rho)$. Then, the equations of motion are given relative to the barycenter; the independent variable is the mean anomaly l

$$\begin{aligned}
&\ddot{x} - 2(1+v)\dot{y} - \dot{v}y - (1+v)^2 x + \frac{1-\mu}{r_1^{\,3}}[x-\mu(1-\rho)] + \\
&\qquad\qquad \frac{\mu}{r_2^{\,3}}[x+(1-\mu)(1+\rho)] = V_x \\
&\ddot{y} + 2(1+v)\dot{x} + \dot{v}x - (1+v)^2 y + y\left(\frac{1-\mu}{r_1^{\,3}} + \frac{\mu}{r_2^{\,3}}\right) = V_y \\
&\ddot{z} + z\left(\frac{1-\mu}{r_1^{\,3}} + \frac{\mu}{r_2^{\,3}}\right) = V_z
\end{aligned} \tag{2}$$

where $r_1^{\,2} = [x-\mu(1+\rho)]^2 + y^2 + z^2$, $r_2^{\,2} = [x+(1-\mu)(1+\rho)]^2 + y^2 + z^2$, and V_x, V_y, V_z are components of perturbing acceleration on the third body (direct effect).

The indirect effect then is incorporated through the perturbation quantities ρ, v, by means of the equations

$$\ddot{r} - r\dot{\theta}^2 = -r^{-2} + \partial R/\partial r; \qquad r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)\partial R/\partial \theta \tag{3}$$

where R is the disturbing function on the motion of the primaries. Thus, it is necessary to integrate Eqs. (3), i.e., to solve analytically a perturbed two-body problem, in order to exhibit complete equations of motion for the perturbed problem. Moreover, even in the absence of perturbations, the effects of eccentric orbits of the primaries appear in Eqs. (2) as-if they were perturbations. Thus, it is of interest whether the solution of Eqs. (3) can be avoided, in deriving equations of motion.

An alternate approach follows from the derivation of equations of motion for the elliptic restricted problem by means of the Nechvile transformation.[3] In this derivation, the equations of motion in siderial coordinates are transformed and exhibit explicitly the left-hand sides of Eqs. (3) [see Eq. (61), p. 592 of Ref. 3]. Thus, let coordinates ξ, η, ζ be normalized with respect to variable r, f = true anomaly is the independent variable, $\tilde{r}_j^{\,2} = (\xi-\xi_j)^2 + \eta^2 + \zeta^2$, $j = 1, 2$, and $\xi_1 = -(1-\mu)$, $\xi_2 = \mu$. Also, α is defined by

$$\begin{aligned}
r(df/dl)^2 &= (1+\alpha)(1+e\cos f)/r^2 = (1+e\cos f)/r^2 + \\
&\qquad 2(1+e\cos f)^{1/2}\left[\textstyle\int (\partial R/\partial\theta)\,dl\right]/r^{5/2} + \left[\textstyle\int (\partial R/\partial\theta)\,dl\right]^2/r^3
\end{aligned}$$

and the equations then are given

$$\begin{aligned}
&\xi'' - 2\eta' - [(1+\alpha)(1+e\cos f)]^{-1}\Bigg[\xi\left(1 - r^2\frac{\partial R}{\partial r}\right) - \\
&\qquad \frac{(1-\mu)(\xi-\mu)}{\tilde{r}_1^{\,3}} - \frac{\mu(\xi+1-\mu)}{\tilde{r}_2^{\,3}} - (\xi'+\eta)r\frac{\partial R}{\partial\theta}\Bigg] = \\
&\qquad\qquad V_\xi \cdot r^2[(1+\alpha)(1+e\cos f)]^{-1} \\
&\eta'' + 2\xi' - [(1+\alpha)(1+e\cos f)]^{-1}\Bigg[\eta\left(1 - r^2\frac{\partial R}{\partial r} - \frac{1-\mu}{\tilde{r}_1^{\,3}} - \frac{\mu}{\tilde{r}_2^{\,3}}\right) - \\
&\qquad (\eta'-\xi)r\frac{\partial R}{\partial\theta}\Bigg] = V_\eta \cdot r^2[(1+\alpha)(1+e\cos f)]^{-1} \\
&\zeta'' + \zeta\left[1 - [(1+\alpha)(1+e\cos f)]^{-1}\left(1 - r^2\frac{\partial R}{\partial r} - \frac{1-\mu}{\tilde{r}_1^{\,3}} - \frac{\mu}{\tilde{r}_2^{\,3}}\right)\right] + \\
&\qquad \zeta' r\frac{\partial R}{\partial\theta} = V_\zeta \cdot r^2[(1+\alpha)(1+e\cos f)]^{-1}
\end{aligned} \tag{4}$$

In Eqs. (4), f and r appear explicitly, as in Eq. (2). Hence, for an exact treatment, one must again solve Eqs. (3), in order to obtain complete equations of motion. In such a situation, Eqs. (2) are simpler and hence preferable to Eqs. (4). But in Eqs. (4), wherever f and r appear, they are multiplied by perturbation quantities involving e, R, or V. Hence, in an approximate treatment wherein one is willing to neglect terms of order R^2, eR, etc., f and r can be given by the usual two-body relations and solution of Eqs. (3) is not required. Also α can then be neglected. Then, in such a treatment, one can use the disturbing function directly, avoiding the need for a perturbed two-body solution in deriving equations of motion.

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Wall Shear in Strongly Retarded and Separated Compressible Turbulent Boundary Layers

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Nomenclature

A^+ = measure of sublayer thickness
$\bar{p}$ = mean static pressure
P^+ = dimensionless pressure gradient, $(v/\rho u_\tau^{\,3})\,d\bar{p}/dx$

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